

Paring instability in the mixed state of a d -wave superconductor

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We propose that an excitonic gap can be generated along nodal directions by Coulomb interaction in the mixed state of d -wave cuprate superconductors. In a superconductor, the Coulomb interaction usually cannot generate any fermion gap since its strength is weakened by superfluidity. It becomes stronger as superfluid density is suppressed by external magnetic field, and is able to generate a gap for initially gapless nodal quasiparticles beyond some critical field H_c . By solving the gap equation, it is found that the nodal gap increases with growing field H , which leads to a suppression of thermal conductivity at zero temperature. This mechanism naturally produces the field-induced thermal metal-insulator transition observed in transport experiments.

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I. INTRODUCTION

The low-energy spectral and transport properties of nodal quasiparticles in d -wave cuprate superconductor are very important issues. In the absence of external magnetic field, the ground state is occupied by uniform superconductivity. For a clean superconductor, the low-energy density of states vanishes linearly as $N(\omega) \sim |\omega|$ upon approaching the Fermi surface. It acquires a finite value at zero energy in the presence of weak impurity scattering. In this case, the nodal quasiparticles exhibit universal transport behavior in the sense that the electric, thermal, and spin conductivities are independent of impurity scattering rate at the $\omega \rightarrow 0$, $T \rightarrow 0$ limit.¹⁻³ The zero-temperature thermal conductivity κ is of particular interests since it is not affected by vertex corrections.² Remarkably, the predicted universal thermal conductivity has been confirmed by heat transport measurements at optimal doping.^{4,5}

When placed in an external perpendicular magnetic field, the superconductor enters into the mixed state in the range $H_{c1} < H < H_{c2}$. Inside the vortex cores, the superfluid current is significantly reduced by the magnetic field. The low-energy fermionic excitations in the mixed state are expected to have rather different low-energy behaviors comparing with those in the uniform zero-field condensate.³ As revealed by heat transport measurements, the thermal conductivity loses its universality and depends on the impurity scattering rate.³ In addition, on the underdoping side, it decreases as the magnetic field grows up.^{6,7} There seems to be a field-induced thermal metal-to-insulator transition in some underdoped cuprate superconductors.^{6,7} These experimental results can be intuitively understood by assuming that the nodal fermions acquire finite mass gap in the mixed state. A phenomenological expression for the nodal gap was proposed⁸ to understand the field-induced reduction in thermal conductivity. However, the dynamic origin for the gap generation has not been discussed.

The goal of this paper is to suggest a mechanism for opening the field-induced gap for the initially gapless nodal quasiparticles. Generally, this mechanism would be realized by some kind of fermion self-interaction or boson-mediated interaction. Such interaction should have the following two

features: it is weak enough to be irrelevant in the uniform superconducting state; it gets stronger with growing perpendicular magnetic field so that a finite gap is generated beyond some critical magnetic field H_c .

Qualitatively, the U(1) gauge fluctuation arising from strong correlation provides a good candidate for such mechanism. The t - J model of cuprate superconductors can be theoretically treated by the slave boson method. After making mean-field analysis and including fluctuations, there appears an emergent U(1) gauge field which interacts strongly with spin-carrying spinons and charge-carrying holons.⁹ The superconductivity is realized by holon condensation below T_c , while the d -wave energy gap is formed by spinon pairing. In the superconducting state, the low-energy elementary excitations are gapless nodal spinons and the U(1) gauge boson is gapped via the Anderson-Higgs mechanism. The finite gauge boson gap weakens gauge interaction, so usually no fermion gap can be generated. However, once the superfluid density is suppressed by external magnetic field, the gap of U(1) gauge boson decreases and the strength of gauge interaction increases with growing magnetic field.¹⁰ Then a finite gap for nodal spinons could be generated by gauge fluctuation, leading to suppression of thermal conductivity. Unfortunately, it is hard to average over the vortex distributions within this formalism due to the complexity brought by spin-charge separation.

The gapless nodal fermions might acquire a gap via the magnetic catalyst mechanism¹¹ when they are placed in an external magnetic field. But this mechanism depends on a crucial assumption that the fermion stays in the lowest Landau level.^{11,12} However, in the case of high-temperature superconductor, the Landau level has been shown not to be the appropriate description of fermion energy spectrum in the mixed state.¹³ Therefore, the magnetic catalyst mechanism is unlikely to be at work.

In this paper, we study the possibility of gap generation due to the long-range Coulomb interaction between charged nodal quasiparticles. Two quasiparticles that carry the same charges always experience a repulsive Coulomb force, while the quasiparticle and quasihole experience an attractive Coulomb force. When the attractive force is sufficiently strong, it is possible that a Dirac quasiparticle is combined with a Dirac quasihole to form a stable excitonic pair. Through this

mechanism, the gapless fermion acquires a finite excitonic gap.

Recently, this kind of gap generation was argued to lead to an insulating ground state in single layer graphene, when the Coulomb interaction strength g is larger than some threshold g_c and the fermion flavor N is less some threshold N_c .^{14–16} Moreover, an interesting superfluidity was predicted to exist in bilayer graphene based on a similar pairing instability.¹⁷ The long-range Coulomb interaction is also very important in cuprate superconductors. First of all, it lifts the gapless Goldstone mode up to plasmon mode, which is actually the rudiment of Higgs mechanism. Its importance in the formation of stripe phase has been emphasized by several authors.¹⁸ However, its role and influence on nodal quasiparticles are still in debate.¹⁹ In the absence of a reliable microscopic theory of Coulomb interaction, we resort to the phenomenological approach.

From the available extensive experiments, we know that the nodal quasiparticles have rather long mean-free path and behave like well-defined Bogoliubov-Landau quasiparticles in the uniform superconducting state.^{20,21} This fact and its excellent agreement with analysis of the conventional Bardeen-Cooper-Schrieffer (BCS) type^{1,2,20} implies that the Coulomb interaction must be fairly weak in the superconducting state and generally cannot generate any fermion gap, except in the lightly doping region. On the other hand, in the nonsuperconducting ground state, it is generally believed that there are no well-defined Landau quasiparticles. It is reasonable to expect that the long-range Coulomb interaction is very strong in this state. The field-induced mixed state lies between these two extreme limiting cases. As the superfluid density decreases with magnetic field, the effective strength of Coulomb interaction gets stronger. For sufficiently strong interaction, a dynamical fermion gap can be generated by forming excitonic pairs. To implement this intuitive picture with (explicit) computations, we assume a phenomenological form for the effective interaction strength which is a function of magnetic field H . After solving the associated gap equation, we find that the growing magnetic field drives the system toward a phase transition into excitonic insulating state beyond some critical value H_c . Once the nodal fermion acquires a finite gap m , the low-energy fermionic excitations are significantly suppressed below the scale m , leading to reduction in thermal conductivity. (This can help to understand the transport behaviors observed in the mixed state of cuprate superconductors.)

Besides the thermal metal-insulator transition, another important issue about field-induced phenomena is the enhancement of antiferromagnetic correlations inside the vortex cores. Such microscopic coexistence of magnetic order and superconductivity has been investigated experimentally^{22–28} and theoretically.^{29,30} From the field-theoretic point of view, the field-induced antiferromagnetism or spin-density wave can be represented by a mass term for nodal fermions.^{8,31} With this identification, the mechanism responsible for the thermal metal-insulator transition can also account for the existence of field-induced magnetic order in the mixed state.

Before presenting the technical details, we would like to point out that a number of assumptions and approximations will be used to simplify discussions on the issue of dynamical

gap generation. Thus, the conclusions reached in this paper are reliable only at the qualitative, rather than quantitative, level.

The paper is arranged as follows. In Sec. II, we build the model and write down the gap equation. In Sec. III, we propose the phenomenological form of the effective Coulomb interaction and calculate its dependence on magnetic field H . In Sec. IV, we solve the gap equation and give the field dependence of critical coupling g_c and dynamical gap. The qualitative understanding of transport experiments is also discussed. We ends with a summary and discussion in Sec. V.

II. MODEL HAMILTONIAN AND GAP EQUATION

We begin our discussion with the following Hamiltonian of d -wave superconductor:

$$H_0 = \sum_{\mathbf{k}} \Phi_{\mathbf{k}}^\dagger [\epsilon_{\mathbf{k}} \tau_3 - \Delta_{\mathbf{k}} \tau_1] \Phi_{\mathbf{k}}, \quad (1)$$

where the standard two-component Nambu spinor representation $\Phi_{\mathbf{k}}^\dagger = (c_{\mathbf{k}\uparrow}^\dagger, c_{-\mathbf{k}\downarrow})$ is adopted and τ_i is Pauli matrix. The electron dispersion is $\epsilon_{\mathbf{k}} = -2t(\cos k_x a + \cos k_y a) - \mu_0$ with μ_0 being the chemical potential and the d -wave energy gap is $\Delta_{\mathbf{k}} = \frac{\Delta_0}{2}(\cos k_x a - \cos k_y a)$. The quasiparticle spectrum is $E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$, which has four nodal points at the Fermi level. Linearizing the dispersion in the vicinity of the nodes, one obtains the spectrum $E_{\mathbf{k}} = \sqrt{v_F^2 k_1^2 + v_\Delta^2 k_2^2}$, where \mathbf{k}_1 is perpendicular to the Fermi surface and \mathbf{k}_2 is parallel to the Fermi surface. The four-component Dirac spinor can be defined as^{8,32,33}

$$\Psi_{1(2)}^\dagger(\mathbf{q}, \omega_n) = \{c_{\uparrow}^\dagger(\mathbf{k}, \omega_n), c_{\downarrow}(-\mathbf{k}, -\omega_n), \\ c_{\uparrow}^\dagger[\mathbf{k} - \mathbf{Q}_{1(2)}, \omega_n], c_{\downarrow}[-\mathbf{k} + \mathbf{Q}_{1(2)}, -\omega_n]\},$$

where $\mathbf{Q}_{1(2)} = 2\mathbf{K}_{1(2)}$ is the wave vector that connects the nodes within the diagonal pairs, $\mathbf{k} = \mathbf{K}_i + \mathbf{q}$ with $\mathbf{q} \ll \mathbf{K}_i$. Here, we use the four-component spinor because it is impossible to define chiral symmetry in two-component representation of fermion field in $(2+1)$ dimensions.

The continuum Hamiltonian of free Dirac fermions can be written as

$$H_0 = i \int d^2\mathbf{r} \bar{\Psi}_1 (\gamma_1 v_F \partial_x + \gamma_2 v_\Delta \partial_y) \Psi_1 + (1 \rightarrow 2, x \leftrightarrow y), \quad (2)$$

where $\bar{\Psi} = \Psi^\dagger \gamma_0$. The 4×4 matrices can be chosen as $\gamma_0 = \sigma_1 \otimes \tau_0$, $\gamma_1 = -i\sigma_2 \otimes \tau_3$, and $\gamma_2 = i\sigma_2 \otimes \tau_1$, where σ_i acts in the subspace of the nodes in a diagonal pair and τ_i acts on indices inside a Nambu field. There are two matrices anti-commuting with them, $\gamma_3 = i\sigma_2 \otimes \tau_2$ and $\gamma_5 = \sigma_3 \otimes \tau_0$. The matrices satisfy the Dirac algebra $\{\gamma_\mu, \gamma_\nu\} = 2 \text{diag}(1, -1, -1)$.

The Hamiltonian for the Coulomb interaction is

$$H_C = \frac{1}{4\pi} \sum_{i,i'=1}^N \int_{\mathbf{r},\mathbf{r}'} \bar{\Psi}_i(\mathbf{r}) \gamma_0 \Psi_i(\mathbf{r}) \frac{g}{|\mathbf{r}-\mathbf{r}'|} \bar{\Psi}_{i'}(\mathbf{r}') \gamma_0 \Psi_{i'}(\mathbf{r}'). \quad (3)$$

The bare Coulomb interaction in momentum space is $V_0(q) = g/|\mathbf{q}|$. The parameter g measures the strength of bare Coulomb interaction in the nonsuperconducting ground state.³⁴ It is easy to see that this model is very similar to that in single-layer graphene, where the low-energy excitations are also massless Dirac fermions. Unlike the semimetal background in graphene, in the present case the Dirac fermions are in a charge condensate, which unavoidably affect their properties and the Coulomb interaction between them.

From the results obtained in graphene, it is known that sufficiently strong Coulomb interaction can lead to finite excitonic gap, when the fermion flavor is below a critical value.¹⁴⁻¹⁶ We speculate that a similar pairing instability to occur in the mixed state of d -wave superconductor. When studying the gap generation, we fix the physical fermion flavor $N=2$. Thus, coupling g becomes the only variable that tunes the excitonic phase transition.

The Hamiltonian is invariant under the continuous chiral transformation $\Psi \rightarrow e^{i\theta\gamma_3(s)}\Psi$. It will be dynamically broken once the Dirac fermion acquires a finite mass via the effective Coulomb interaction. This phenomenon is nonperturbative in nature and generally can be studied by analyzing the self-consistent Dyson equation

$$G^{-1}(p) = G_0^{-1}(p) + T \sum_{n=-\infty}^{\infty} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \gamma_0 G(k) \Gamma_0(p, k) V(p-k), \quad (4)$$

where the $(2+1)$ -dimensional momentum is defined as

$$k = (i\omega_n, \mathbf{k}). \quad (5)$$

The Matsubara frequency is $\omega_n = (2n+1)\pi T$ for fermions and $\omega_n = 2n\pi T$ for bosons. Here, $\Gamma_0(p, k)$ is the full vertex function. The free propagator for massless Dirac fermion is

$$G_0(k) = \frac{1}{i\omega_n \gamma_0 - v_F k_1 \gamma_1 - v_\Delta k_2 \gamma_2}. \quad (6)$$

Due to the Coulomb interaction, it becomes the complete propagator $G(p)$, which is determined by the Dyson equation. In the case of $(2+1)$ -dimensional quantum electrodynamics (QED₃), the dynamical chiral symmetry breaking can be most conveniently studied using the $1/N$ expansion.³⁵ Here we follow the same strategy and keep only the leading order of $1/N$ expansion. So we can neglect the wavefunction renormalization and replace the vertex function Γ_0 by γ_0 . Now the complete propagator can be formally written as

$$G(k) = \frac{1}{i\omega_n \gamma_0 - v_F k_1 \gamma_1 - v_\Delta k_2 \gamma_2 - m(k)}, \quad (7)$$

where $m(k)$ denotes the Dirac fermion mass. Further, as shown in the context of QED₃, at least at low energies and to the leading order of $1/N$ expansion, the velocity anisotropy

is irrelevant to the critical behavior.³⁶ We simply set $v_F = v_\Delta = 1$ whenever they multiply the momenta (k_1, k_2) in the gap equation.

The full Coulomb interaction function is

$$V(q) = \frac{1}{|\mathbf{q}|/g + N\chi(q)}. \quad (8)$$

The polarization function $\chi(q)$ contains all information about how the Dirac fermions response to the many-particle system. We first consider the nonsuperconducting ground state. Within the random-phase approximation, the fermion propagator G and the vertex function Γ_0 are both replaced by the bare ones, i.e.,

$$\chi(q) = -T \sum_{n=-\infty}^{\infty} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \text{Tr}[\gamma_0 G_0(k) \gamma_0 G_0(k-q)]. \quad (9)$$

Inserting the expression for the interaction function, the gap equation can now be written as

$$m(p) = T \sum_{n=-\infty}^{\infty} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{m(k)}{\omega_n^2 + \mathbf{k}^2 + m^2(k)} \frac{1}{|\mathbf{q}|/g + N\chi(q)}, \quad (10)$$

where $q=p-k$. In the instantaneous approximation, $\chi(q)$ has the following zero-frequency expression:³⁷

$$\chi(\mathbf{q}) = \frac{2T}{\pi} \int_0^1 dx \log \left[2 \cosh \frac{\sqrt{x(1-x)}|\mathbf{q}|}{2T} \right]. \quad (11)$$

Now the gap m is independent of frequency and the frequency summation can be carried out with the result

$$m(T, \mathbf{p}) = \int \frac{d^2\mathbf{k}}{8\pi^2} \frac{m(T, \mathbf{k})}{\sqrt{\mathbf{k}^2 + m^2(T, \mathbf{k})}} \frac{\tanh \frac{\sqrt{\mathbf{k}^2 + m^2(T, \mathbf{k})}}{2T}}{|\mathbf{q}|/g + N\chi(\mathbf{q})}. \quad (12)$$

In the limit of zero temperature, $\chi(\mathbf{q}) = |\mathbf{q}|/8$, the gap equation further simplifies to

$$m(\mathbf{p}) = \int \frac{d^2\mathbf{k}}{8\pi^2} \frac{m(\mathbf{k})}{\sqrt{\mathbf{k}^2 + m^2(\mathbf{k})}} \frac{1}{|\mathbf{q}|/g + N|\mathbf{q}|/8}. \quad (13)$$

The nontrivial solution $m(\mathbf{p})$ of this integral equation signals the occurrence of dynamical mass generation.

When the ground state is occupied by the uniform superconductivity, the Coulomb interaction function must be modified. In the mixed state, the superfluid density is a non-uniform quantity which has different values at different spatial positions. To study the gap equation in the mixed state, we should average over the vortices and obtain a mean value of superfluid density. This is the task of the next section.

III. ANSATZ OF EFFECTIVE INTERACTION

In the underdoping and optimal doping regions, the ground state is occupied by the superconductivity, which significantly weakens the Coulomb interaction between Dirac

fermions. Such effect can be described by calculating the polarization function that incorporates the effect of finite superfluid density. However, it is not clear how to correctly calculate the polarization function in the superconducting state, so we will assume a phenomenological form for the effective interaction function. From the experimental facts, we know that the interaction must reach its minimal value when the superfluid density Λ_s takes its maximal value. As Λ_s decreases with growing magnetic field H , the effective interaction strength increases and eventually takes its maximal value after the superconductivity is completely destroyed. We assume the following *ansatz* for the effective interaction function in the superconducting state:

$$V(q, H) = \frac{1}{|\mathbf{q}|/g + N\chi(q)} \frac{1}{1 + \alpha\Lambda_s(H)}, \quad (14)$$

where α is an adjustable parameter. This is the simplest function that can describe the reduction in strength by superfluid density. It is largest at the limit $\Lambda_s=0$, and is smallest at the limit $H=0$. The function $[1 + NV_0(q)\chi(q)][1 + \alpha\Lambda_s(H)]$ can be considered as the effective dielectric function of superconducting state. In the mixed state, the field-dependent superfluid density $\Lambda_s(H)$ controls the effective strength of Coulomb interaction. The parameter α must be properly chosen so that a moderately strong magnetic field H_c separates the gapless and gaped phases. In order to see how the critical point depends on H , we need to solve the gap equations after including $\Lambda_s(H)$.

To study the gap equation, the superfluid density $\Lambda_s(H)$ should be obtained by averaging over the vortices. In the mixed state, the low-energy properties of d -wave superconductor are dominated by the extended quasiparticles in the bulk material, unlike the case of conventional s -wave superconductor. Volovik³⁸ proposed a semiclassical approach and showed that the density of states varies as \sqrt{H} at low temperatures, which has been observed by experiments.³ Within the semiclassical treatment, the effects of circulating supercurrent around vortices can be represented by a Doppler shift³⁹⁻⁴¹ in the quasiparticle spectrum, $\omega \rightarrow \omega + \mathbf{k} \cdot \mathbf{v}_s(\mathbf{r})$, where $\mathbf{v}_s(\mathbf{r})$ is the superfluid velocity at a position \mathbf{r} and \mathbf{k} is the quasiparticle momentum which can be approximated by its value at the node. Then the fermion Green's function can be written as $G(\omega, \mathbf{k}, \mathbf{r}) = G[\omega + \epsilon_i(\mathbf{r}), \mathbf{k}]$, where $\epsilon_i(\mathbf{r}) = \mathbf{k}_i \cdot \mathbf{v}_s(\mathbf{r})$. The local value $F(\mathbf{r})$ of any physical quantity F determined by the Green's function can be obtained using the above local Green's function. The field-dependent quantity $F(H)$ is written as the following spatial average $F(H) = \frac{1}{A} \int d^2\mathbf{r} F[\omega + \epsilon(\mathbf{r})]$, where the integral is taken over a unit cell of the vortex lattice with area A . Such averaging integral depends on the vortex distribution. The field-dependent quantity is

$$F(\omega, H) = \int_{-\infty}^{\infty} d\epsilon F(\omega + \epsilon) \mathcal{P}(\epsilon), \quad (15)$$

with probability function $\mathcal{P}(\epsilon) = \frac{1}{A} \int d^2\mathbf{r} \delta[\epsilon - \mathbf{k} \cdot \mathbf{v}_s(\mathbf{r})]$.

There are several possible choices of $\mathcal{P}(\epsilon)$, which were discussed in Ref. 41. For example, the distribution function of vortex liquid or solid is $\mathcal{P}(\epsilon) = \frac{E_H^2}{2(\epsilon^2 + E_H^2)^{3/2}}$; for disordered

vortex state, it takes the form $\mathcal{P}(\epsilon) = \frac{1}{\sqrt{\pi}E_H} \exp(-\frac{\epsilon^2}{E_H^2})$. The typical energy scale of Doppler shift is $E_H = \frac{v_F}{2R} = \frac{v_F}{2} \sqrt{\frac{\pi H}{\Phi_0}}$, where $R = (\Phi_0/\pi H)^{1/2}$ is the radius of the unit cell of vortex lattice and $\Phi_0 = hc/2e$ is one quantum of magnetic flux. The field dependence of a physical quantity, such as density of state or specific heat, depend somewhat on the choices of distribution function, but the qualitative result is not sensitive to the choice.

The computation of superfluid density $\Lambda_s(H)$ within the semiclassical approximation has already been performed in Ref. 33, so we just list the basic steps and cite the results. The superfluid stiffness is given by^{2,33}

$$\Lambda_s^{ij}(T, H) = \tau^{ij} - \Lambda_n^{ij}(T, H), \quad (16)$$

where τ_{ij} is the diamagnetic tensor and Λ_n represents the normal-fluid density divided by the carrier mass. In the Matsubara formalism, the normal-fluid density is³³

$$\Lambda_n^{ij} = -T \sum_{n=-\infty}^{\infty} \int_{\text{HBZ}} \frac{d^2\mathbf{k}}{(2\pi)^2} v_F^i(\mathbf{k}) v_F^j(\mathbf{k}), \quad (17)$$

$$\text{Tr}[G(i\omega_n, \mathbf{k}) \gamma_0 \gamma_5 G(i\omega_n, \mathbf{k}) \gamma_0 \gamma_5],$$

with the i -component velocity v_F^i . Here, HBZ means the halved Brillouin zone of the d -wave superconductors, i.e., the domain with two neighboring nodes. The averaged normal-fluid density is given by³³

$$\Lambda_n^{ij}(H) = \int_{-\infty}^{\infty} d\epsilon \mathcal{P}(\epsilon) \int_{\text{HBZ}} \frac{d^2\mathbf{k}}{(2\pi)^2} \int_{-\infty}^{\infty} d\omega \tanh \frac{\omega}{2T} \frac{v_F^i v_F^j}{4i\pi}$$

$$\times \text{Tr}[G_A(\omega - \epsilon, \mathbf{k}) \gamma_0 \gamma_5 G_A(\omega - \epsilon, \mathbf{k}) \gamma_0 \gamma_5$$

$$- G_R(\omega - \epsilon, \mathbf{k}) \gamma_0 \gamma_5 G_R(\omega - \epsilon, \mathbf{k}) \gamma_0 \gamma_5]. \quad (18)$$

In principle, the superfluid density should be calculated by including the complete fermion propagators $G_{R,A}(\omega, \mathbf{k})$ into Eq. (18). Hence it actually satisfies an equation that couples self-consistently to the gap equation. It is not an easy task to solve these coupled equations in practice. However, there is a remarkable simplification if we are mainly interested in what happens in the vicinity of the critical point of chiral phase transition. Near the bifurcation point, the gap equation can be linearized and the gap appearing in the superfluid density can be taken to be zero. In the limit $m \rightarrow 0$, the normal-fluid density finally becomes³³

$$\Lambda_n(H) = \frac{v_F}{2\pi v_\Delta} \int_{-\infty}^{\infty} d\epsilon \mathcal{P}(\epsilon) J(m, \epsilon), \quad (19)$$

with $J(\epsilon) = 2|\epsilon|$. If we adopt the distribution function $\mathcal{P}(\epsilon)$ of vortex liquid, then the superfluid density

$$\Lambda_s(H) = \tau - \frac{v_F}{\pi v_\Delta} E_H. \quad (20)$$

Here, τ is the zero-temperature superfluid density in the absence of magnetic field and the ratio between v_F and v_Δ appears as a coefficient. Although the anisotropy is irrelevant when $v_{F(\Delta)}$ multiplies particle momenta, the ratio might be

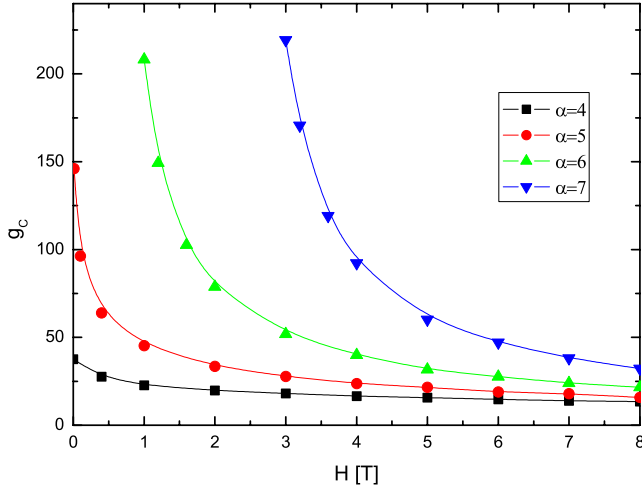


FIG. 1. (Color online) The dependence of critical strength g_c on magnetic field H for several choices of α .

important in this expression. We simply set the ratio $v_F/v_\Delta = 20$ in the following discussions.

IV. FIELD DEPENDENCE OF CRITICAL COUPLING AND THERMAL CONDUCTIVITY

After getting the effective Coulomb interaction, we now study the self-consistent gap equation

$$m(\mathbf{p}) = \int \frac{d^2\mathbf{k}}{8\pi^2} \frac{m(\mathbf{k})}{\sqrt{\mathbf{k}^2 + m^2(\mathbf{k})}} \frac{1}{|\mathbf{q}|/g + N|\mathbf{q}|/8} \frac{1}{1 + \alpha\Lambda_s(H)}. \quad (21)$$

This integral equation can be solved numerically by the parameter embedding method, with the Coulomb interaction strength g being the turning parameter. There is a critical value g_c that separates the chiral-symmetric phase ($m=0$ for $g < g_c$) from the symmetry-breaking phase ($m \neq 0$ for $g > g_c$). g_c is just the critical point of chiral phase transition. We can see that excitonic pairing is quite different from conventional BCS-type pairing: the former is produced only by sufficiently strong, attractive Coulomb force between particles and holes, while the latter is triggered by arbitrary weak attractive force between electrons. The excitonic gap breaks the chiral symmetry and leads to the formation of antiferromagnetism or spin-density wave in the field-induced vortex state.

Following Ref. 33, the zero-temperature superfluid density is taken to be $\tau=1500$ K and the energy scale $E_H \sim 30\sqrt{HKT}^{-1/2}$. The parameter α is a variable (in unit of eV^{-1}) depending on doping concentration and the type of cuprate superconductors. It surely is not a universal quantity and cannot be uniquely determined. For completeness, we consider a number of possible values, $\alpha=4, 5, 6, 7$. For each value of α , the relationship between critical strength g_c and magnetic field H is shown in Fig. 1. As it turns out, critical strength g_c decreases as the magnetic field H grows up.

To judge whether an excitonic gap is generated for nodal fermions, we can simply compare the physical strength g

with the critical value g_c . Admittedly the exact value of physical strength g in the nonsuperconducting ground state is unknown. However, we can make a simple comparison between its value in cuprate superconductors with that in single layer graphene. In graphene, the typical value g is about ~ 20 . It is not unreasonable to estimate that the parameter g in the nonsuperconducting ground state be larger than 20 since the correlation is known to be very strong in cuprate superconductors.

If we assume that $g=50$, then the critical field $H_c = 0, 1.0, 3.5, 6.0$ for parameters $\alpha=4, 5, 6, 7$, respectively. For $\alpha=4$, a finite excitonic gap is generated even in zero-field case. This is able to explain several experimental results performed in some underdoped cuprates in the absence of external field H : the finite nodal gap found by photoemission,⁴² the suppression of thermal conductivity κ from the universal value with lowering doping concentration,⁴³ and the coexistence of competing magnetic order with superconductivity.²⁷ For larger values $\alpha=5, 6, 7$, the gap is generated only for magnetic field H larger than some critical value H_c . These parameters are relevant to the doping regions in which the nodal gap and competing order appear only in the field-induced state.^{22–26} It appears that the phenomenological parameter α should be an increasing function of doping concentration. The cases for other values of g and α can be analyzed similarly.

The generated gap will surely affect all observable physical quantities, such as specific heat, electric, and thermal conductivities. Here we are primarily interested in the zero-temperature thermal conductivity κ . If we assume a constant fermion gap m and a small impurity scattering rate Γ_{imp} , then the zero-temperature thermal conductivity has the expression⁸

$$\frac{\kappa}{T} \propto \frac{\Gamma_{\text{imp}}^2}{\Gamma_{\text{imp}}^2 + m^2}. \quad (22)$$

It is no longer universal and depends explicitly on impurity scattering rate as well as on gap m . Obviously, in the weak impurity limit, $\Gamma_{\text{imp}} \ll m$, the thermal conductivity is rapidly suppressed from its universal value by m . To see how κ varies with magnetic field H , we need to know the field dependence of gap $m(H)$. To this end, we solved the gap Eq. (21) numerically⁴⁴ and presented the results for $\alpha=4$ and $g=200$ in Fig. 2. It is evident that the gap $m(H)$ is an increasing function of magnetic field H . Thus in the symmetry broken phase, as magnetic field H grows, the thermal conductivity is suppressed and the system undergoes a phase transition from thermal metal to thermal insulator at high-field limit, which is qualitatively in agreement with transport experiments.^{6,7}

As revealed by transport experiments, the field-induced reduction in thermal conductivity occurs only in underdoped and optimally doped cuprate superconductors.^{6,7} In the overdoped region, the thermal conductivity is found to increase with growing magnetic field H .^{6,7} This can be understood by assuming that the d -wave superconductivity responses differently to external magnetic field in underdoped and overdoped cuprates. On the underdoping side, the magnetic field

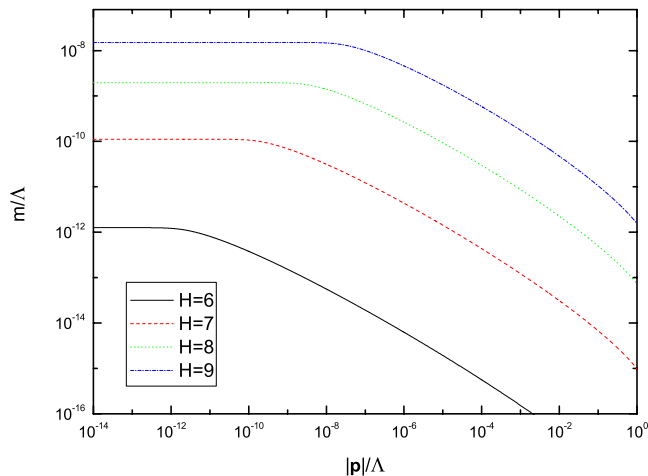


FIG. 2. (Color online) The dependence of the generated gap on momentum and magnetic field for $\alpha=4$ and $g=200$. Λ is the momentum cutoff.

only reduces the superfluid density inside vortex cores but leaves the d -wave energy gap essentially unchanged. Due to the additional excitonic gap along nodal directions generated by Coulomb interaction, the low-energy nodal quasiparticles are significantly suppressed, thus reducing the thermal conductivity. However, on the overdoping side, the magnetic field destroys the d -wave energy gap by directly breaking the Cooper pairs. As a result, extended quasiparticles are excited from the condensate and the thermal conductivity increases with growing field H .

V. SUMMARY AND DISCUSSION

In summary, we proposed a mechanism to explain the field-induced reduction in thermal conductivity in the mixed state of d -wave cuprate superconductor. In this mechanism, a finite gap for nodal fermions is generated by the strong Coulomb interaction between gapless particle and hole excitations. Since the Coulomb interaction is usually weak in the superconducting state, such gap generation is possible only after the superfluid density is reduced by strong external magnetic field. The excitonic gap reduces the thermal con-

ductivity at low temperature, so there is a thermal metal-insulator transition driven by magnetic field.

There are several effects that might change the critical behavior of chiral phase transition. For example, the long-range Coulomb interaction can be screened by the finite zero-energy density of states produced by disorder scattering and/or vortex scattering. Such screening effect reduces the possibility of fermion gap generation.¹⁶ Only in clean superconductor and at magnetic field much lower than the up critical field H_{c2} this effect can be ignored. On the other hand, the gap generation can also be promoted by other mechanisms. If there are strong contact interaction between nodal fermions, the possibility of pairing instability is significantly enhanced due to the positive contribution from contact interaction.¹⁶ These competing effects can be included into the above calculations along the steps presented in Ref. 16.

Throughout the present paper, the thermal fluctuation effect is totally omitted. This effect actually plays at least three important roles. First, the thermal fluctuation effectively excites quasiparticles out of the condensate and hence reduces the superfluid density rapidly. Second, these thermally excited quasiparticles lead to screening of the Coulomb interaction. In addition, the excitonic pairing will surely be suppressed by thermal fluctuations. These effects compete with each other, making the situation rather complicated. This is why we simply neglect the thermal effects and consider only nearly zero temperature. To make an extension to finite temperatures, all these three effects should be carefully analyzed.

Finally, we must admit that in the present work we utilized a number of assumptions and approximations when studying the dynamical gap generation for Dirac fermions. The results obtained in this paper are only qualitatively reliable. In particular, we have not arrived at a quantitative determination of the critical magnetic field H_c . Generally speaking, H_c must be a function of doping concentration, temperature, and type of superconductor sample. Unfortunately, it is difficult to quantitatively include any of these effects. We wish the present work will be put on a firmer theoretical ground in the future.

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